CERN-TH.7208/94

The next-to-leading QCD approximation to the Ellis-Jaffe sum rule

S.A. Larin [1]

Theory Division, CERN, CH - 1211, Geneva 23, Switzerland

Abstract

The α_s^2 correction to the Ellis-Jaffe sum rule for the structure function g_1 of polarized deep inelastic lepton-nucleon scattering is calculated.

CERN-TH.7208/94
March 1994

[1]Permanent address: INR, Moscow 117312, Russia.

The results of the EMC Collaboration at CERN [1] and E130 Collaboration at SLAC [2] for the Ellis-Jaffe [3] sum rule $\int_0^1 dx g_1^p(x,Q^2)$ attracted a lot of attention to this sum rule; see [4]-[12] and references therein. Recent data of the SMC Collaboration at CERN [13] on polarized scattering of muons off deuterium and of the E142 Collaboration at SLAC [14] on polarized scattering of electrons off helium ^3He allowed the determination of the analogous sum rule $\int_0^1 dx g_1^n(x,Q^2)$ for a neutron. This in turn allowed us to find a difference $\int_0^1 dx[g_1^p(x,Q^2) - g_1^n(x,Q^2)]$ which is the Bjorken sum rule [15]. At present, the Bjorken sum rule is calculated within QCD with quite high accuracy. The α_s correction [16], α_s^2 correction [17] and α_s^3 correction [18] are calculated in the leading twist approximation. The higher twist corrections are also calculated [19]. For the Ellis-Jaffe sum rule only the α_s correction was calculated in the leading twist [20]. The power corrections were calculated in [21].

In the present paper we obtain the α_s^2 correction to the Ellis-Jaffe sum rule in the leading twist massless quark approximation. All calculations are performed in dimensional regularization [22]. Renormalizations are done within the $\overline{MS}$-scheme [23], the standard modification of the Minimal Subtraction scheme [24].

Polarized deep inelastic electron-nucleon scattering is described by the hadronic tensor

$$W_{\mu\nu}(p,q) = \frac{1}{4\pi}\int d^4z\ e^{iqz} < p,s \mid J_\mu(z)J_\nu(0) \mid p,s >=$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right)\frac{1}{p\cdot q}F_2(x,Q^2)+$$

$$+ i\varepsilon_{\mu\nu\rho\sigma}q_\rho\left[\frac{s_\sigma}{p\cdot q}g_1(x,Q^2) + \frac{s_\sigma p\cdot q - p_\sigma q\cdot s}{(p\cdot q)^2}g_2(x,Q^2)\right], \qquad (1)$$

of which we will consider the structure function g_1. Here $J_\mu = \overline{\psi}\gamma_\mu\hat{E}\psi = \sum_{i=1}^{n_f} e_i\overline{\psi}_i\gamma_\mu\psi_i$ is the electromagnetic quark current and $\hat{E} = diag(2/3,-1/3,-1/3,...)$ is the quark electromagnetic charge matrix. $x = \frac{Q^2}{2p\cdot q}$ is the Bjorken variable, $Q^2 = -q^2$. The nucleon state $\mid p,s >$ is covariantly normalized as $< p,s \mid p',s' >= \delta_{ss'}2p^0(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')$. s_σ is the polarization vector of the nucleon: $s_\sigma = \overline{U}(p,s)\gamma_\sigma\gamma_5U(p,s)$, where U is the nucleon spinor, $\overline{U}(p,s)U(p,s) = 2M$.

The moments of the deep inelastic structure functions are expressed [25] via quantities of the Wilson operator product expansion (OPE) of the corresponding currents. We need the OPE of two electromagnetic currents. The strict method of the OPE ensures [26],[27] that the OPE of two gauge-invariant currents can contain only gauge-invariant operators with their renormalization basis. Thus we have only contributions from the non-singlet and singlet axial currents in the OPE of electromagnetic currents in the leading twist for the considered structure:

$$i\int dz\ e^{iqz}T\{J_\mu(z)J_\nu(0)\} \stackrel{Q^2\to\infty}{=}$$

$$= \varepsilon_{\mu\nu\rho\sigma} \frac{q_\rho}{q^2} \left[\sum_a C^a(\log(\frac{\mu^2}{Q^2}), a_s(\mu^2)) J^{5,a}_\sigma(0) + C^s(\log(\frac{\mu^2}{Q^2}), a_s(\mu^2)) J^5_\sigma(0) + higher\ twists \right], \qquad (2)$$

where the non-singlet contribution can be rewritten as

$$\sum_a C^a J^{5,a}_\sigma(0) = C^{ns} \sum_a Tr(\hat{E}^2 t^a) J^{5,a}_\sigma(0),$$

to introduce, as usual, the unique non-singlet coefficient function C^{ns} not depending on the number a.

Here $J^{5,a}_\sigma(x) = \overline{\psi}(x)\gamma_\sigma\gamma_5 t^a \psi(x)$ is the non-singlet axial current, where t^a is a generator of a flavour group, $Tr(t^a t^b) = \frac{1}{2}\delta^{ab}$, and $J^5_\sigma(x) = \sum_{i=1}^{n_f} \overline{\psi}_i(x)\gamma_\sigma\gamma_5\psi_i(x)$ is the singlet axial current. The known twist-two and spin-one axial gluon current $K_\sigma = 4\varepsilon_{\sigma\nu_1\nu_2\nu_3}(A^a_{\nu_1}\partial_{\nu_2}A^a_{\nu_3} + \frac{1}{3}g f^{abc} A^a_{\nu_1} A^b_{\nu_2} A^c_{\nu_3})$ has also the necessary quantum numbers, but it cannot contribute to the above operator product expansion because it is not gauge invariant. Here and further on (before presenting the final results), we use the most practical definition for the strong coupling constant from the calculational point of view

$$a_s = \frac{g^2}{16\pi^2} = \frac{\alpha_s}{4\pi}.$$

The Ellis-Jaffe sum rule is expressed as

$$\int_0^1 dx g_1^{p(n)}(x, Q^2) = C^{ns}(1, a_s(Q^2)) \left(\pm \frac{1}{12}|g_A| + \frac{1}{36} a_8 \right) +$$

$$+ C^s(1, a_s(Q^2)) \exp\left(\int_{a_s(\mu^2)}^{a_s(Q^2)} da'_s \frac{\gamma^s(a'_s)}{\beta(a'_s)} \right) \frac{1}{9}\Sigma(\mu^2), \qquad (3)$$

where some comments are in order. Here $p(n)$ denotes a target: proton (or neutron). The plus (minus) before $|g_A|$ corresponds to the proton (neutron) target. The proton matrix elements of the axial currents are defined as follows:

$$|g_A| s_\sigma = 2 < p, s \mid J^{5,3}_\sigma \mid p, s >= (\Delta u - \Delta d) s_\sigma,$$

where $g_A/g_V = -1.2573 \pm 0.0028$ [28] is the constant of the neutron beta-decay;

$$a_8 s_\sigma = 2\sqrt{3} < p, s \mid J^{5,8}_\sigma \mid p, s >= (\Delta u + \Delta d - 2\Delta s) s_\sigma,$$

$$\Sigma(\mu^2) s_\sigma =< p, s \mid J^5_\sigma \mid p, s >= (\Delta u + \Delta d + \Delta s) s_\sigma,$$

and we use the standard notation

$$\Delta q(\mu^2) s_\sigma =< p, s \mid \overline{q}\gamma_\sigma\gamma_5 q \mid p, s >, \quad q = u, d, s.$$

We omitted the contributions of the nucleon matrix elements for quarks heavier than the s-quark but it is straightforward to include them. We should stress here that g_A and a_8 do

not depend on the renormalization point μ^2 since the corresponding non-singlet currents $J^{5,3}_\sigma$ and $J^{5,8}_\sigma$ are conserved in the massless limit, and hence their renormalization constants are equal to one. On the contrary, the singlet axial current has a non-trivial renormalization constant. Hence the quantity $\Sigma(\mu^2)$ does depend on the renormalization point (that is why it is not a physical quantity). The coefficient functions $C^{ns}(1, a_s(Q^2))$ and $C^s(1, a_s(Q^2))$ are normalized in the standard way to the unity at the tree level. The renormalization group technique was applied to the coefficient functions to kill logarithms $\log(\frac{\mu^2}{Q^2})$.

The singlet axial current has the non-zero anomalous dimension $\gamma^s(a_s)$ due to the axial anomaly [29, 30]. So one can say that the axial anomaly contributes to this sum rule through the renormalization group exponent of the singlet current contribution in eq.(3).

Let us define the functions $\gamma^s(a_s)$ and $\beta(a_s)$ in the renormalization group exponent of eq.(3). The renormalization group QCD β-function is calculated [31, 32] in the $\overline{MS}$-scheme at the 3-loop level :

$$\beta(a_s) = \mu^2 \frac{da_s}{d\mu^2} = -\beta_0 a_s^2 - -\beta_1 a_s^3 - -\beta_2 a_s^4$$
$$= -\left(11 - \frac{2}{3} n_f\right) a_s^2 - \left(102 - \frac{38}{3} n_f\right) a_s^3 - \left(\frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2\right) a_s^4. \quad (4)$$

To define the anomalous dimension of the singlet axial current we should first define the singlet axial current itself within dimensional regularization. To define the singlet axial current we will follow the lines of ref. [33], where the 't Hooft -Veltman definition [22] of the γ_5-matrix is elaborated for the multiloop case. The singlet current J^5_σ is renormalized multiplicatively and is expressed via the bare one $[J^5_\sigma]_B$ as

$$J^5_\sigma = Z^s_5 Z^s_{MS} [J^5_\sigma]_B. \quad (5)$$

Here Z^s_{MS} is the $\overline{MS}$ renormalization constant which contains only poles in the regularization parameter ϵ, the dimension of the space-time being $D = 4 - 2\epsilon$. The extra finite renormalization constant Z^s_5 is introduced to keep the exact 1-loop Adler-Bardeen form [34] for the operator anomaly equation within dimensional regularization in all orders in a_s:

$$\partial_\mu J^5_\mu = a_s \frac{n_f}{2} (G\tilde{G}), \quad (6)$$

where all quantities are renormalized ones. $G\tilde{G} = \varepsilon_{\mu\nu\lambda\rho} G^a_{\mu\nu} G^a_{\lambda\rho}$ and $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$ is the gluonic field strength tensor.

In fact the full physical quantity, the Ellis-Jaffe sum rule, does not depend on the choice of the normalization constant Z^s_5. But to be definite we adopt the normalization of [33] to keep in the $\overline{MS}$-scheme the singlet axial current satisfying eq.(6).

The anomalous dimension of the singlet axial current is zero at the 1-loop level and starts from the 2-loop level. To have the next-to-leading approximation we need two non-zero terms, i.e. the 3-loop approximation. The 3-loop approximation for the anomalous

dimension of the singlet axial current was calculated in [33] and confirmed in [35]. The result in the adopted normalization reads

$$\gamma^s(a_s) = \mu^2 \frac{d\log(Z_5^s Z_{MS}^s)}{d\mu^2} = \gamma^{(0)} a_s + \gamma^{(1)} a_s^2 + \gamma^{(2)} a_s^3 =$$

$$= a_s^2(-6C_F n_f) + a_s^3 \left[\left(18C_F^2 - \frac{142}{3} C_F C_A \right) n_f + \frac{4}{3} C_F n_f^2 \right]. \tag{7}$$

Here $C_F = \frac{4}{3}$ and $C_A = 3$ are the Casimir operators of the fundamental and adjoint representation of the colour group $SU(3)$. The a_s^2 term agrees with the calculation [20] after multiplication of our result by the factor (-2) due to different normalizations.

The non-singlet coefficient function $C^{ns}(1, a_s(Q^2))$ was calculated in the a_s^3 approximation in [18] where the Bjorken sum rule was calculated in this approximation. We want to obtain the a_s^2 correction to the singlet contribution to the Ellis-Jaffe sum rule (3):

$$C^s(1, a_s(Q^2)) \exp\left(\int_{a_s(\mu^2)}^{a_s(Q^2)} da_s' \, \frac{\gamma^s(a_s')}{\beta(a_s')} \right) \frac{1}{9} \Sigma(\mu^2) =$$

$$= C^s(1, a_s(Q^2)) \left[1 - a_s(Q^2) \frac{\gamma^{(1)}}{\beta_0} + a_s(Q^2)^2 \frac{\gamma^{(1)}\beta_1 + (\gamma^{(1)})^2 - \gamma^{(2)}\beta_0}{2\beta_0^2} \right] \frac{1}{9} \Sigma_{inv}, \tag{8}$$

where we introduced the notation

$$\Sigma_{inv} \equiv \exp\left(- \int^{a_s(\mu^2)} da_s' \, \frac{\gamma^s(a_s')}{\beta(a_s')} \right) \Sigma(\mu^2) \tag{9}$$

for the renormalization group-invariant (i.e. μ^2-independent) nucleon matrix element of the singlet axial current.

Beside the 3-loop (the order a_s^3) approximation of the anomalous dimension, we need also the 2-loop (the order a_s^2) approximation for the singlet coefficient function $C^s(1, a_s)$ which has already been calculated in [36]. Here we present the calculation of C^s with another method to confirm the validity of the result obtained in [36]. We use the "method of projectors" [37]. To project out the coefficient function C^s from the OPE of eq.(2), one should sandwich this equation between quark states and nullify the quark momentum p. To be more precise, one should consider the following Green function

$$i \int dz \; e^{iqz} < 0 \mid T\overline{\psi}(p)\gamma_\sigma\gamma_5\psi(p) J_\mu(z) J_\nu(0) \mid 0 >|_{p=0}^{amputated} =$$

$$= \varepsilon_{\mu\nu\rho\sigma} \frac{q_\rho}{q^2} C^s(\log(\frac{\mu^2}{Q^2}), a_s(\mu^2)) < o \mid T\overline{\psi}(p)\gamma_\sigma\gamma_5\psi(p) Z_5^s Z_{MS}^s [J_\sigma^5(0))]_B \mid 0 >|_{p=0}^{amputated}, \tag{10}$$

where some remarks are in order. $\psi(p)$ is the Fourier transform of the quark field carrying the momentum p. Quark legs are amputated. The essence of the method [37] is the nullification of the quark momentum p. In the dimensional regularization scheme all massless vacuum diagrams are equal to zero. So on the r.h.s. only the tree graphs survive after the nullification

of p. In our case the only operator which produces a tree graph is J^5_σ. The non-singlet axial current does not contribute because of the nullification of the flavour trace: $Tr(t^a) = 0$. It is interesting to note that at $p = 0$ we have infrared divergences in the diagrams of the l.h.s. of eq.(10). But these divergences are cancelled by the ultraviolet poles of the renormalization constant Z^s_{MS} of the singlet current.

Thus to calculate C^s we need to calculate the diagrams contributing to the l.h.s. of eq.(10). These are the diagrams of the forward scattering of a photon off quarks with photon momentum q and zero quark momentum. In comparison with the calculation of the non-singlet coefficient function C^{ns} [17, 18], we have at the 2-loop level two extra diagrams where both electromagnetic vertices are inside a closed quark loop. The analytic calculation of the diagrams has been done with the symbolic manipulation program FORM [38] by means of the package MINCER [39]. This package is based on algorithms of ref. [40]. The result is

$$C^s(1, a_s(Q^2)) = 1 + a_s\left(-3C_F\right) + a_s^2\left[\frac{21}{2}C_F^2 - 23C_FC_A + \left(8\zeta_3 + \frac{13}{3}\right)C_Fn_f\right], \tag{11}$$

where ζ_3 is the Riemann zeta-function ($\zeta_3 = 1.202056903\ldots$). This result of ours agrees with the calculation in [36] if one takes into account the fact that another finite constant Z^{ns}_5 (relevant for the non-singlet axial current, see [18, 33]) was taken in [36] for the normalization of the singlet axial current instead of our Z^s_5. Multiplying our result (11) by the factor

$$\frac{Z^s_5}{Z^{ns}_5} = \frac{1 + a_s(-4C_F) + a_s^2(22C_F^2 - \frac{107}{9}C_AC_F + \frac{31}{18}C_Fn_f)}{1 + a(-4C_F) + a^2(22C_F^2 - \frac{107}{9}C_FC_A + \frac{2}{9}C_Fn_f)} + O(a_s^3)$$

one can reproduce the result of [36]. So we have a strong check of C^s.

In principle our technique allows us to compute also the α_s^3 correction to the singlet coefficient function C^s. But we need then also the 4-loop anomalous dimension of the singlet axial current in order to take into account this correction self-consistently in the next-next-to-leading approximation for the the Ellis-Jaffe sum rule. The calculation of the 4-loop anomalous dimension is very time-consuming at present, although all the necessary techniques are available.

Collecting together all the relevant results for the coefficient functions and the anomalous dimension, we obtain finally the next-to-leading approximation for the Ellis-Jaffe sum rule:

$$\int_0^1 dx g_1^{p(n)}(x, Q^2) = \left\{1 - \left(\frac{\alpha_s(Q^2)}{\pi}\right) + \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\left(-\frac{55}{12} + \frac{1}{3}n_f\right) + \right.$$

$$\left. + \left(\frac{\alpha_s(Q^2)}{\pi}\right)^3\left[-\frac{13841}{216} - \frac{44}{9}\zeta_3 + \frac{55}{2}\zeta_5 + \left(\frac{10339}{1296} + \frac{61}{54}\zeta_3 - \frac{5}{3}\zeta_5\right)n_f - \frac{115}{648}n_f^2\right]\right\}\times$$

$$\times\left(\pm\frac{1}{12}|g_A| + \frac{1}{36}a_8\right) +$$

$$+\left\{1-\left(\frac{\alpha_s(Q^2)}{\pi}\right)+\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\left[-\frac{55}{12}+n_f\left(\frac{13}{36}+\frac{2}{3}\zeta_3\right)\right]\right\}\times$$

$$\times\left[1+\left(\frac{\alpha_s(Q^2)}{\pi}\right)\frac{6n_f}{(33-2n_f)}+\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\frac{\frac{1029}{4}n_f+\frac{23}{2}n_f^2+\frac{1}{3}n_f^3}{(33-2n_f)^2}\right]\times$$

$$\times\exp\left(-\int^{a_s(\mu^2)}da_s'\frac{\gamma^s(a_s')}{\beta(a_s')}\right)\frac{1}{9}\Sigma(\mu^2). \tag{12}$$

Here we use $\frac{\alpha_s}{\pi}=\frac{g^2}{4\pi^2}$ for the strong coupling constant. We keep the known (extra for the next-to-leading approximation) α_s^3 term [18] for the non-singlet part, since this part determines the Bjorken sum rule. For the singlet part we factorize the Q^2-dependent factors coming from the coefficient function (the first factor in the singlet part) and from the renormalization group exponent (the second factor). The leading α_s term agrees with [20].

For the case $n_f=3$ the sum rule reads

$$\int_0^1 dx g_1^{p(n)}(x,Q^2)=\left[1-\left(\frac{\alpha_s(Q^2)}{\pi}\right)-3.5833\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2-20.2153\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3\right]\times$$

$$\times\left(\pm\frac{1}{12}|g_A|+\frac{1}{36}a_8\right)+$$

$$+\left[1-\left(\frac{\alpha_s(Q^2)}{\pi}\right)-1.0959\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\right]\left[1+0.6666\left(\frac{\alpha_s(Q^2)}{\pi}\right)+1.2130\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\right]\times$$

$$\times\exp\left(-\int^{a_s(\mu^2)}da_s'\frac{\gamma^s(a_s')}{\beta(a_s')}\right)\frac{1}{9}\Sigma(\mu^2). \tag{13}$$

The Ellis-Jaffe sum rule looks most compact for the choice $\mu^2=Q^2$ since the renormalization group exponent becomes a unity. But then the Q^2-dependence jumps inside the matrix element of the singlet axial current Σ:

$$\int_0^1 dx g_1^{p(n)}(x,Q^2)=\left[1-\left(\frac{\alpha_s(Q^2)}{\pi}\right)-3.5833\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2-20.2153\left(\frac{\alpha_s(Q^2)}{\pi}\right)^3\right]\times$$

$$\times\left(\pm\frac{1}{12}|g_A|+\frac{1}{36}a_8\right)+$$

$$+\left[1-\left(\frac{\alpha_s(Q^2)}{\pi}\right)-1.0959\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\right]\frac{1}{9}\Sigma(Q^2). \tag{14}$$

The conclusion is that the α_s^2 correction to the Ellis-Jaffe sum rule is quite small in the $\overline{MS}$-scheme.

Acknowledgements

I am grateful to K.G. Chetyrkin, J. Ellis, B.L. Ioffe, M. Karliner, W.L. van Neerven and J.A.M. Vermaseren for helpful discussions. I would like to thank the Theory Division of CERN for warm hospitality. The work is supported in part by the Russian Fund of the Fundamental Research, Grant N 94-02-04548-a.

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